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Interest rate corridors and reserves ☆

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Abstract

This paper evaluates reserves regimes versus interest rate corridors, which have become competing frameworks for monetary policy implementation. Rate corridors, relying on lending and deposit facilities to create ceilings and floors for overnight interest rates, evince mixed results on controlling volatility. Reserve requirements allow period-average smoothing of interest rates but, even if remunerated, are subject to reserve avoidance activities. A system of voluntary, period-average reserve commitments could offer equivalent rate-smoothing advantages. If central banks created symmetric opportunity costs of meeting or falling short of period-average reserve requirements (or commitments), they could achieve flat reserve demand on settlement day.
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1. Key characteristics of current policy implementation regimes

Since the 1980s, central banks have generally moved away from implementing monetary policy through targeted control of narrow measures of money in favor of targets for overnight interest rates. Over the last half-decade or so, a healthy competition has arisen between two types of regimes for controlling overnight interest rates, a reserve regime and

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a rate corridor. Each employs open market operations to try to hit the targeted interest rate, but they differ in the institutional structure within which such operations are conducted. Enough time has passed now to warrant evaluations of these alternative policy implementation systems.

A *reserve regime*, as at the Federal Reserve and the Bank of Japan, for instance, uses the traditional tool of reserve requirements that must be met on an average basis over a maintenance period of several weeks. Reserve requirements are useful for either monetary or interest rate control. For control of a narrow monetary aggregate, reserve requirements against contemporaneous measures of deposits are most helpful. For control of an overnight interest rate, however, lagged reserve requirements are preferable because they give the central bank and market participants advance information about the aggregate demand for reserves over a maintenance period. The Federal Reserve switched from contemporaneous to lagged reserve requirements in August 1998 largely to improve interest rate control. The Fed's Open Market Desk now can plan its open market operations to provide a sufficient average base of reserves to enable banks to meet their known requirements over a two-week maintenance period. The Desk then makes small day-by-day adjustments to its program to offset the effects of temporary demand pressures (such as distortions in the distribution of reserves arising from Social Security payments or settlement of Treasury securities) and of fluctuations in Fed balance sheet items (such as currency flows, Treasury deposits, and float). The period-averaging feature of reserve requirements not only improves the predictability of average reserve demand over a maintenance period, but also gives banks an incentive to arbitrage the federal funds rate across the days of the period, helping to keep it on target.

A pure *rate corridor regime*, as currently practiced in Australia, Canada, New Zealand, Sweden, and Switzerland, relies on standing facilities, rather than period-average reserve requirements, to help achieve a target interest rate. A ceiling for interest rates is provided by a lending facility at a penalty interest rate above the central bank's target rate, while a floor is provided by the interest rate on balances held at the central bank. With a fairly narrow spread between ceiling and floor rates, centered on the policy target, the regime is generally called an interest rate channel, tunnel, or corridor. To be helpful for the implementation of policy, as shown in the model developed below, it is crucial to have symmetric opportunity costs on short or long positions on accounts at the central bank. Thus, Australia, Canada, and New Zealand all lend at 25 basis points above their target for overnight rates and pay interest at 25 basis points below the policy target. Overnight overdrafts are automatically booked as loans from the central bank at the standard lending rate. Switzerland employs a spread of 50 basis points above and below its target for three-month Libor, while Sweden uses a spread of 75 basis points above and below its target repo rate. In late June, 2001, the Bank of England (BOE) instituted a deposit facility which, combined with its traditional lending facility, completed a rate corridor framework with a spread of 100 basis points on each side of its policy target. (The BOE also has a small reserve requirement to raise revenue for its operating expenses, but without any period averaging, as banks must meet the requirement on a daily basis.)

Although nearly all central banks have a lending facility, and thus an ability to establish an interest rate ceiling, such facilities often have not been combined with an adjustable interest rate floor and thus fail to establish the essential symmetric opportunity costs of a rate corridor regime. The Federal Reserve, for instance, has been unable to obtain legislative authorization to pay interest on reserves. Moreover, until recently, the Fed had no clear interest rate ceiling, as important non-pecuniary penalties and stigma effects that could differ from bank to bank obscured the true costs of discount window borrowing. In early 2003, however, the Fed changed its discount window by eliminating administrative discouragement of borrowing and establishing an explicit penalty interest rate (at 1 percentage point above the target funds rate for most banks). With that new discount facility and a zero floor interest rate on reserves, the Fed inadvertently had the symmetric penalties of a rate corridor from June 2003 to June 2004 when the funds rate target was 1 percent.

Below, a pure rate corridor model is analyzed and related to the experience of countries using such a system. A hybrid system is then depicted which employs both reserve requirements and a rate corridor, as implicitly practiced by the Fed during the above period, and as employed by the European Central Bank (ECB), also with a spread of 1 percentage point above and below its policy target. A third model is then developed to display a regime of voluntary reserve commitments, which has some features that will be incorporated in a new regime to be implemented by the BOE and which is also reflective of the contractual clearing balance program of the Federal Reserve. In the Fed's program, banks contract to hold a minimum level of period-average "clearing balances" that earn implicit interest in the form of credits against Fed service charges. The pre-commitment and period-averaging features of these contractual balances give them the same predictability and arbitrage benefits for policy implementation as reserve requirements, and the program engenders no incentives for banks to engage in the tax avoidance activities associated with reserve requirements.

2. A rate corridor regime

In the absence of period-average reserve requirements, a rate corridor regime can be represented by a one-day model. Some features of this regime have been previously described in Clinton (1997), Guthrie and Wright (2000), and Woodford (2001). The model below also builds on such earlier literature as Orr and Mellon (1961), Poole (1968), Ho and Saunders (1985), Campbell (1987), Spindt and Hoffmeister (1988), and Kopecky and Tucker (1993), along with more recent contributions from Clouse and Dow (1999), Furfine (2000), Bindseil (2000), Bartolini et al. (2001, 2002), and Heller and Lengwiler (2003).

In a rate corridor regime, the central bank relies on lending and deposit facilities to create symmetric opportunity costs around its target overnight rate. A private bank[1] tries to track its account position with the central bank during the day, but is nevertheless subject to unexpected late payments or delayed accounting information, and therefore can determine its end-of-day position only within a margin of error given by a stochastic term, ε, where $E(\varepsilon) = 0$.[2] During the day, the bank can trade central bank balances with other banks at the market rate, i, and does so to achieve a target account balance of T. If the bank's actual end-of-day account balance of $T+\varepsilon$ is positive, it earns interest at the central

[1]The term "private bank" is used to represent any institution that may open a central bank account and participate in the payment system supported by the central bank.

[2]This over-simplifies actual information flows and market frictions late in the day, at least in the US. Account uncertainty is likely reduced to fairly low levels for most banks by the end of trading. However, the shocks modeled here could reflect unexpected late-day payments that occur even before the market closes, if the bank is unable to borrow or lend in sufficient quantities in the late-day market to offset such shocks owing to line-limit constraints.

bank's deposit interest rate. Denoting the central bank's desired overnight rate as i^*, other assumptions are:

(A1) A representative, competitive bank is risk-neutral at the margin.

(A2) Loans are freely available from the central bank, as perfect substitutes for borrowings from the private market, at an interest rate of i^*+s. End-of-day account overdrafts are booked as loans at that interest rate.

(A3) Balances left overnight in an account at the central bank are perfect substitutes for lending in private markets and earn interest at the rate of i^*-s.

When choosing its target account balance, T, the private bank's information set is: $I_0 = \{i^*, s, i$, and the distribution of the account balance shock, $F(\varepsilon)$, with $E(\varepsilon) = 0\}$.

If the bank had full information, it would set $T = -\varepsilon$, given $i^*-s<i<i^*+s$. Then its end-of-day balance at the central bank would be zero and its funding cost would be minimized at $-i\varepsilon$. Without knowing ε, the bank chooses T to minimize two types of expected costs: the opportunity cost of holding a positive end-of-day balance in its account at the central bank, relative to lending funds in the market, given by $i-(i^*-s)$, and the loss, in the case of overdrafts, on borrowing from the central bank rather than from the market, given by i^*+s-i. Formally, the bank's problem is:

$$\min_{T} {}_{I_0} \int_{-T}^{\infty} (i - i^* + s)(T + \varepsilon)\mathrm{d}F(\varepsilon) - \int_{-\infty}^{-T} (i^* + s - i)(T + \varepsilon)\mathrm{d}F(\varepsilon). \tag{1}$$

The first-order condition may be written in the form:

$$F(-T^*) = \frac{1}{2} + \frac{i - i^*}{2s}, \tag{2}$$

where T^* is the optimal choice. Interpretations are facilitated by assuming either:

(A4) The distribution of the account shock, $F(\cdot)$, is symmetric, or

(A4′) The account shock has a normal distribution with variance σ^2.

The assumption of normality is a natural one to make for the representative bank because the central limit theorem implies that, as the number of banks becomes large, the sum of account shocks across banks and thus the aggregated demand for balances has a normal distribution. In the period when the Federal Reserve implicitly operated a corridor system (June 2003–June 2004), a Jarque-Bera statistic of 4.1 on daily excess reserves indicates that the null hypothesis of normality cannot be rejected.[3] However, the Federal Reserve has about 8,000 account-holders; the normality assumption may not hold as well for central banks that have a small number of accounts.

If the market interest rate equals the central bank's desired rate, then Eq. (2) indicates that $F(-T^*) = 1/2$, and for a zero-mean symmetric distribution, (A4), this can occur only if banks target a zero balance ($T^* = 0$), as noted by Woodford (2001). This reflects a key advantage of a rate corridor regime: the zero-balance target at $i = i^*$ is unaffected by changes in i^* or in the spread, s, as long as the policy target remains at the midpoint between the central bank's lending and deposit rates.

[3]It is not possible to isolate late-in-the-day account shocks from other events affecting reserve positions at the aggregate level, such as lumpy supply adjustments by the Fed. The latter, however, would most likely bias the result away from normality. The test did exclude a few days around the New York black-out event of August 2003 and a one-day tax-related spike in reserves in April 2004.

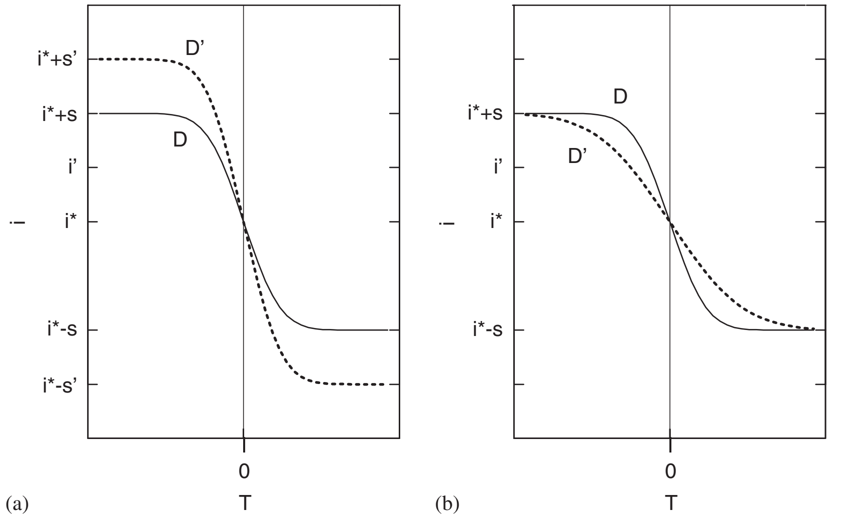

Fig. 1. Simple rate corridor model. Demand for central bank balances: (a) increase in spread, and (b) increase in uncertainty.

Under (A4′), $F(-T^*) = N(-T^*/\sigma)$, where $N(\cdot)$ is the cumulative standard normal distribution function, and the comparative statics are

$$\frac{\partial T^*}{\partial i} = \frac{-1}{2sn(T^*/\sigma)} < 0 \quad \text{and} \quad \frac{\partial T^*}{\partial s} = \frac{1 - 2N(T^*/\sigma)}{2sn(T^*/\sigma)}, \tag{3}$$

with $n(\cdot)$ denoting the standard normal density. It is readily shown that the elasticity of T^* with respect to σ is unity and also that a narrower spread implies, from Eq. (3), a flatter demand curve near the target interest rate (where $T^* = 0$).

Fig. 1 plots reserve demand with a normal distribution of account shocks, showing the greater elasticity with a narrower spread or increased account uncertainty.[4] With the vertical intercept evident as a pivot point in both cases, the advantage of symmetric opportunity costs is revealed: The account uncertainty perceived by a private bank, σ, would vary across banks and over time for the same bank. As individual bank account uncertainty and the mix of banks in the market varied from day to day, the shape of the aggregate reserve demand curve would change. Nevertheless, if the central bank kept its target interest rate at the midpoint of the corridor, the aggregate quantity of non-borrowed reserves needed to hit the target rate would be constant over time at zero. By contrast, a central bank that set its target at another point, such as i' in Fig. 1, would have to re-estimate the demand curve day-by-day to determine the quantity of reserves needed to achieve the target rate.

2.1. The distribution of market rates

Of course, arranging for the aggregate supply of non-borrowed reserves to equal zero on a daily basis is also a non-trivial task. The Federal Reserve devotes considerable effort to

[4] Poole (1968) first pointed out the heightened elasticity of reserve demand owing to a wider dispersion in the distribution of account shocks.

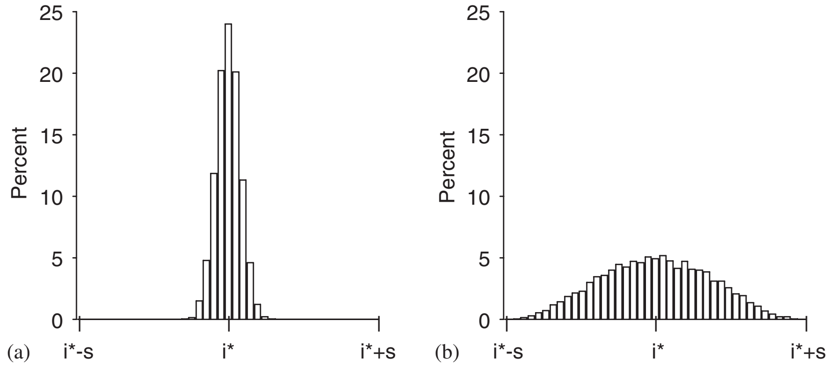

Fig. 2. Simulations of market rates in corridor model: (a) central/private bank uncertainty = 1/10, and (b) central/private bank uncertainty = 1/2. *Note*: Model simulations use 10,000 draws of shocks to the central bank's reserve supply.

predicting daily fluctuations of balance sheet items so that it can offset their reserve effects with open market operations. In the model, the aggregate demand for non-borrowed reserves can be found by inverting Eq. (2) to find the demand for balances of an individual firm, T^*, and then summing across individual banks, indexed by j, to obtain:

$$D = \sum_j T_j^* = -\sum_j F_j^{-1}\left(\frac{1}{2} + \frac{i - i^*}{2s}\right) = -N^{-1}\left(\frac{1}{2} + \frac{i - i^*}{2s}\right)\sum_j \sigma_j,$$

where the last equality follows for a normal distribution.

Now let κ be the central bank's daily error in supplying reserves, where κ has a zero mean and standard deviation σ_κ. Although an increase in private bank account uncertainties, given by $\Sigma\sigma_j$, tends to smooth interest rates by flattening the demand curve as noted above, an increase in reserve supply uncertainty, σ_κ, raises the volatility of interest rates. Fig. 2 depicts histograms of the simulated distribution of market interest rates under different assumptions for the ratio of σ_κ to $\Sigma\sigma_j$ (central/private bank uncertainty). The larger the ratio, the fatter the tails of the resulting distribution of overnight interest rates.

2.2. Weaknesses of a rate corridor regime

A key limitation of a rate corridor framework is the difficulty of ensuring symmetric opportunity costs around the central bank's target rate owing to imperfect substitutability of market transactions for central bank loans. For instance, in the U.S., interbank loans are generally unsecured, but collateral must be posted to borrow from the Fed. Thus, a market rate equivalent to borrowing from the Fed would be $i^* + s$ plus the cost of providing collateral. Any stigma effects in borrowing from the central bank would imply an even higher equivalent market rate. Moreover, the risk-included market rate equivalent to a riskless deposit at the Fed would be $i^* - s$ plus credit risk premium. In such circumstances, a central bank could try to establish symmetry by setting its target rate *above* the midpoint of its corridor or, as at the Bank of Canada (BOC), by supplying a non-zero quantity of aggregate reserves. In the latter case, the quantity of reserves needed to achieve the desired interest rate could vary over time with credit risk perceptions and the opportunity cost of collateral.

A rate corridor system could also be impaired by other frictions. For instance, like the ECB and the BOE, a central bank may pay interest only if a bank shifts funds out of a reserve account and into a special deposit account each day. Moreover, as in the U.K. and Canada, participation in the central bank's standing facilities may be limited to certain institutions, impairing the effectiveness of the ceiling and floor mechanisms. Even in the U.S., where any depository institution may open a Fed account, only 60 percent of those with accounts have completed the borrowing documents needed to access the discount window.

Another disadvantage of a rate corridor regime is that the slope of the demand curve tends to be steepest at the central bank's desired interest rate of i^*. With a symmetric distribution, for instance, the peak slope (at i^*) is $\partial i/\partial T^* = -s$. An elastic demand function is an attractive feature of any implementation regime, as a central bank's reserve supply errors then have little impact on interest rates. Goodfriend (2002) proposed that a central bank could take advantage of the elastic region for interest rates near its deposit rate (evident in Fig. 1) by setting the deposit rate equal to the target interest rate and then supplying enough reserves to drive the market rate to that floor. If private rates were persistently held near one of the boundaries of the corridor, however, trading in the overnight market might dry up; the extent of the consequences for money market infrastructure would depend on whether term instruments were also affected. Possible deleterious effects on market functioning may also militate against increasing the elasticity of demand at the target rate by narrowing the width of the corridor.

2.3. Experience with rate corridors

Woodford (2001) argues that establishment of rate corridor regimes helped Australia, Canada, and New Zealand reduce the volatility of overnight interest rates below that in the United States. Indeed, as shown in Fig. 3(a), Canadian overnight rates have been quite well behaved since the BOC dropped period-average requirements and implemented a pure rate corridor in February 1999. The BOC did find market rates coming in above target in 1999,

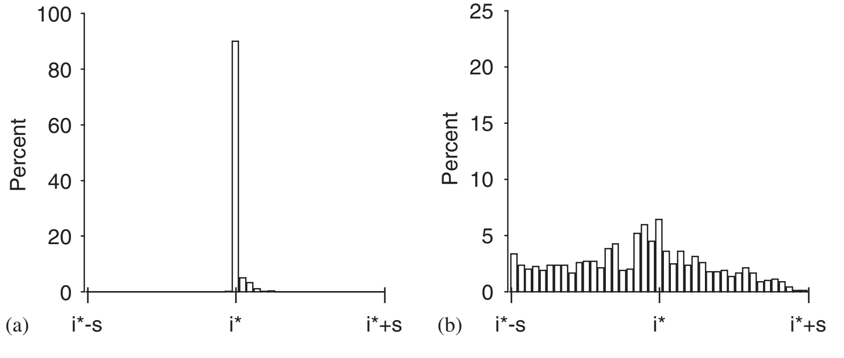

Fig. 3. Market relative to the central bank's target: (a) daily Canadian data, February 1999–December 2004, where i^* varies and $s = .25$, and (b) based on daily SONIA data (BOE), June 2001–November 2004, where i^* varies and $s = 1$.

reflecting implicit asymmetries and incomplete participation in its standing facilities, but performance improved when the BOC began providing a positive level of aggregate reserves (see Bank of Canada, 1999). The distributions of market rates have displayed even tighter supports around central bank targets in Australia and New Zealand after the introduction of rate corridors in June 1998 and March 1999, respectively.

However, as indicated by Fig. 3(b), the experience of the BOE indicates that a rate corridor is not always a panacea. Since the BOE's introduction of a rate corridor in June 2001, the sterling overnight rate has often moved to the boundaries of the corridor, despite a fairly wide spread of 100 basis points on either side of target. Contributing to this result were limitations on participation in standing facilities, a deposit facility that requires daily bank interventions rather than allowing automatic interest payments, overdraft financing at higher rates than 3:30 p.m. central bank loans, and perhaps heightened elasticity and asymmetry issues in a more complex and heterogeneous banking system.

Other central banks operating in complex banking environments have tended to retain period-average reserve requirements. The ECB and, implicitly, the Fed between mid-2003 and mid-2004, have employed both reserve requirements and a rate corridor, a system that is modeled next.

3. A rate corridor with reserve requirements

For analytical tractability, the model developed below assumes a two-day reserve maintenance period. Along with (A1)–(A3), it is assumed that:

(A5) Reserve requirements are assessed against deposits in a previous period and are met by holding a minimum average balance of R at the central bank over a two-day maintenance period. Shortfalls from requirements are made up through borrowing from the central bank at the standard lending rate of i^*+s (or, equivalently, given interest on reserves of i^*-s, through a penalty of $2s$ on the deficiency). Account shocks on settlement day, denoted by ξ, may have a different distribution, $G(\cdot)$, than account shocks on pre-settlement day. The market interest rate and the representative bank's target balance on settlement day are represented by i_S and T_S, respectively, while those on pre-settlement day are denoted by i_P and T_P.

With lagged reserve requirements, account shocks that change the private bank's reservable deposits do not affect its current-period reserve requirement. The model allows settlement day account uncertainty to differ from that on a pre-settlement day, reflecting empirical results of Hamilton (1996) and others. As at the ECB, funds borrowed from the central bank to cover an overdraft can be used to meet reserve requirements (see European Central Bank, 2004). With a minimum daily account balance of zero after borrowing from the central bank, a zero period-average requirement is therefore equivalent to the absence of reserve requirements.

Consider initially the settlement-day problem, assuming that a representative bank has a remaining balance requirement of b. The bank may purchase balances from the market at the settlement day interest rate, i_S, and it earns i^*-s on required or excess balances held at the central bank. It chooses a target balance T_S with the information set: $I_s = \{i^*, s, G(\xi)$, $E(\xi) = 0$, the settlement day market interest rate, i_S, and the remaining required balance as of settlement day, $b\}$.

The bank's settlement day cost minimization problem is then:

$$\min_{T_S} {}_{I_S}(i_S - i^* + s)b + \int_{b-T_S}^{\infty} (i_S - i^* + s)(T_S + \xi - b)\mathrm{d}G(\xi) + \int_{-\infty}^{b-T_S} (i^* + s - i_S)(b - T_S - \xi)\mathrm{d}G(\xi). \quad (4)$$

The first-term is the net cost of borrowing from the market to meet the remaining balance requirement, the first integral is the opportunity cost of holding excess balances, and the last integral is the net cost of borrowings from the central bank, rather than the market, to meet the requirement. If $b = 0$, Eq. (4) becomes equivalent to Eq. (1). Similar to Eq. (2), the first order condition associated with Eq. (4) is

$$G(b - T_S^*) = \frac{1}{2} + \frac{i_S - i^*}{2s}. \quad (5)$$

If $i_S = i^*$ and G is symmetric around the mean of zero, the bank targets its remaining requirement($T_S^* = b$). The plot of the settlement day demand function is merely a rightward shift of the demand function in Fig. 1, with the pivot point at b rather than at zero. The corridor framework thus provides an advantage even in the presence of reserve requirements, as the aggregate demand for *excess* reserves on settlement day pivots around zero at the desired interest rate. In principle, therefore, a central bank using this framework need not form estimates of the settlement day demand curve as its slope changes over time with the account uncertainties perceived by private banks.

To move toward development of the pre-settlement day demand function, note first that $(\partial T_S^*/\partial b) = 1$, or in other words, $b - T_S^*$ is independent of b. From Eq. (5), the optimal value of $b - T_S^*$ can be written as a function of i_S, the distribution G, and other parameters:

$$b - T_S^* \equiv k(i_S, i^*, s, G) = G^{-1}\left(\frac{i_S - i^*}{2s} + \frac{1}{2}\right).$$

Substituting T_S^* into the objective function gives the expected cost function with information set I_S:

$$V(T_S^*|I_S) = (i_S - i^* + s)b + s\int_{k(i_S,i*,s)}^{\infty} \xi \mathrm{d}G(\xi) - s\int_{-\infty}^{k(i_S,i*,s)} \xi \mathrm{d}G(\xi) = (i_S - i^* + s)b + K(i_S, i^*, s, G), \quad (6)$$

where the upper case $K(i_S, i^*, s, G)$ is defined as the last two terms on the first line of Eq. (6). It reflects the expected opportunity cost of having a settlement day balance above or below the remaining requirement with information set I_S.

The pre-settlement day information set, I_p, does not include knowledge of either the remaining balance requirement on settlement day, b, or the settlement day interest rate, i_S: $I_p = \{i_P, i^*, s, F(\varepsilon), G(\xi), \text{and } E(\varepsilon) = E(\xi) = 0\}$.

The value of b is determined after realization of the account shock on pre-settlement day. With the daily average requirement of R and the end-of-day account balance on pre-settlement day of $T_P+\varepsilon$, the value of b is

$$b=\begin{cases} 2R, & \text{if } T_P+\varepsilon\leqslant 0,\\ 2R-T_P-\varepsilon, & \text{if } 0\leqslant T_P+\varepsilon\leqslant 2R,\\ 0, & \text{if } T_P+\varepsilon\geqslant 2R, \end{cases} \quad \text{and} \tag{7}$$

let $e\equiv \mathrm{E}_{I_p}(i_S)-i^*$ represent the private bank's expectation on pre-settlement day of the deviation of the market rate from the central bank's desired interest rate on settlement day. The expectation on pre-settlement day of the opportunity cost of holding balances at the central bank on settlement day is then $e+s$. Ignoring discounting between the first and second day of the maintenance period, and using Eqs. (6) and (7), the pre-settlement day target balance is found from:

$$\begin{aligned}\min_{T_P} {}_{I_P} & \int_{-T_P}^{\infty}(i_p-i^*+s)(T_P+\varepsilon)\mathrm{d}F(\varepsilon)-\int_{-\infty}^{-T_P}(i^*+s-i_p)(T_p+\varepsilon)\mathrm{d}F(\varepsilon)\\ &+2R(e+s)F(-T_P)+(e+s)\int_{-T_P}^{2R-T_P}(2R-T_P-\varepsilon)\mathrm{d}F(\varepsilon)\\ &+\mathrm{E}_{I_p}(K(i_S,i^*,s,G)).\end{aligned} \tag{8}$$

The first line of Eq. (8) is equivalent to Eq. (1). On the second line, the first-term is the expected net cost, with information set I_p, of meeting the period's entire requirement ($2R$) on settlement day, times the probability of having a negative or zero account balance on day one. The next term is the expected net cost of completing the period-average requirement on settlement day if that requirement has been partially met before settlement day. The final term of the equation reflects the expected cost of reserve excesses or deficiencies owing to account shocks on settlement day, but it is unaffected by the pre-settlement day decision.

The first order condition is

$$i_p-i^*+s+(e-s)F(-T_P^*)-(e+s)F(2R-T_P^*)=0. \tag{9}$$

If the pre-settlement day market interest rate is at the central bank's desired rate and expected to be there again on settlement day, so that $i_p=i^*$ and $e=0$, condition (9) simplifies to $1-F(-T_P^*)=F(2R-T_P^*)$, and if F is symmetric around the mean of zero, the bank targets the daily average reserve requirement ($T_P^*=R$) on pre-settlement day. It is easy to show that the comparative statics have the expected signs, with T_P^* responding positively to changes in R and negatively to changes in i_p, as long as the market rate is expected to remain within the corridor ($s>|e|$). Under normality, reserve demand curves for two different expected settlement day interest rates are depicted in Fig. 4(a). Demand is very elastic near the expected settlement day interest rate for a range of values of T_P^* around the daily average requirement. However, because of possible costly overnight overdrafts on either day of the maintenance period, arbitrage is limited and a perfect martingale is not achieved. As shown in Figs. 4(b) and (c), the flat region of the demand curve expands with either a larger reserve requirement or lower account uncertainty.

To examine this region further, condition (9) may be rewritten in the form of a deviation of the pre-settlement day interest rate from the expected interest rate on settlement day:

$$i_P-\mathrm{E}_{I_p}(i_S)=(s-e)N\left(\frac{-T_p^*}{\sigma}\right)+(s+e)\left[N\left(\frac{2R-T_p^*}{\sigma}\right)-1\right], \tag{10}$$

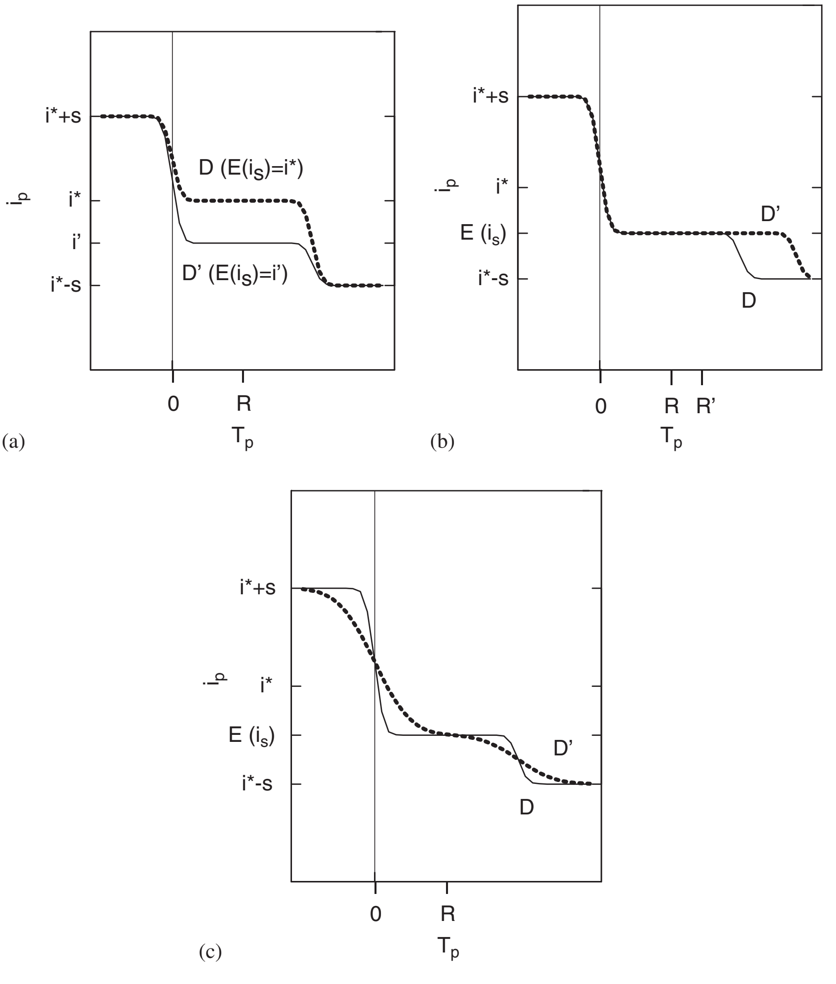

Fig. 4. Time-averaged balance requirements. Demand on pre-settlement day: (a) sensitivity to expected settlement day interest rate, (b) increase in requirements, and (c) increase in uncertainty.

where a normal distribution is assumed. The first term on the right includes the probability of an overdraft on pre-settlement day; it is close to zero when $T^*_P > 2\sigma$. The final bracket is close to zero when there is a low probability of holding excess reserves on settlement day, which is when $T^*_P < 2R - 2\sigma$. The highly elastic region thus occurs where $2\sigma < T^*_P < 2R - 2\sigma$. Intuitively, if the interest rate is near the rate expected on settlement day, risk neutral banks are relatively indifferent as to what proportion of their reserve requirement they satisfy on pre-settlement day. The coefficient on R equals the number of days in the maintenance period, thus suggesting that a longer period would engender a greater range of high elasticity. However, a closed-form solution is not available for longer periods.[5] One argument against long maintenance periods is that a number of banks may

[5]Gaspar et al. (2004) simulate a somewhat similar three-day model, but without analyzing issues regarding the length of the maintenance period. This is a subject for future work.

deliberately run behind on meeting requirements early in the period, expecting interest rates to fall; if these banks run into line limit constraints or other market frictions on settlement days, the volatility of market rates could become more elevated on those days. Moreover, with longer maintenance periods, arbitrage of expected changes in the central bank's target interest rate would be more common.

This model illustrates the advantages of both a corridor system and period-average requirements. The symmetry of opportunity costs of the rate corridor gives the central bank the knowledge that, at the desired interest rate, the aggregate demand for excess reserves on settlement day is zero, irrespective of any time-varying account uncertainties at private banks. If this advantage is not outweighed by the problem of a steep demand curve at that interest rate, combined with imperfect substitutability of market transactions for central bank loans and deposits, the central bank should be aided in achieving its target interest rate on settlement day. If the market expects the interest rate to be on target on settlement day, intertemporal arbitrage induced by period-average requirements should help the same rate to be realized on pre-settlement days. These results abstract from possible non-pecuniary penalties associated with reserve deficiencies and overdrafts, which may vary across banks and over time for the same bank, making it difficult for the central bank to arrange for symmetric opportunity costs.

3.1. Experience with hybrid systems

The ECB employs month-average reserve requirements with a rate corridor of 100 basis points on either side of its policy target. Like the BOE, the ECB has an imperfect floor mechanism in that banks have to transfer funds on any given day to an interest-bearing deposit facility, while excess balances left in the reserve account earn no interest. The Federal Reserve's implicit rate corridor from mid-2003 to mid-2004 also had a 100 basis point spread on either side of its target funds rate (along with its usual two-week maintenance period). Several explicit features of the Fed's framework, including higher charges on overdrafts than discount window loans, impaired the symmetry of a rate corridor.[6] Figs. 5(a) and (b) depict histograms of market rates relative to the central bank's target rates in the U.S. during the above period and in the euro area. Volatility was higher than in Canada (Fig. 3) but smaller than in the U.K. A comparison of the distributions of *intraday* fed funds rates before and during the U.S. rate corridor episode, Figs. 5(c) and (d), suggests that symmetry in official lending and deposit rate spreads added little to the control of volatility. The fairly narrow range of overnight rates achieved by the Fed and the ECB, despite complex financial sectors, thus likely owed importantly to the period-average feature of reserve requirements.

For many years, the Federal Reserve has testified to Congress about the "economically wasteful efforts by depository institutions to circumvent" reserve requirements (Kohn, 2004). Although the reserve tax could be largely eliminated through the payment of a market interest rate on reserves, survey results indicate that banks would in those circumstances nevertheless persist in reserve avoidance activities in order to "earn higher

[6] In addition, borrowings undertaken to bring an overdraft up to a zero balance do not count towards reserve requirements. Reserve deficiency charges also exceed the Fed's lending rate.

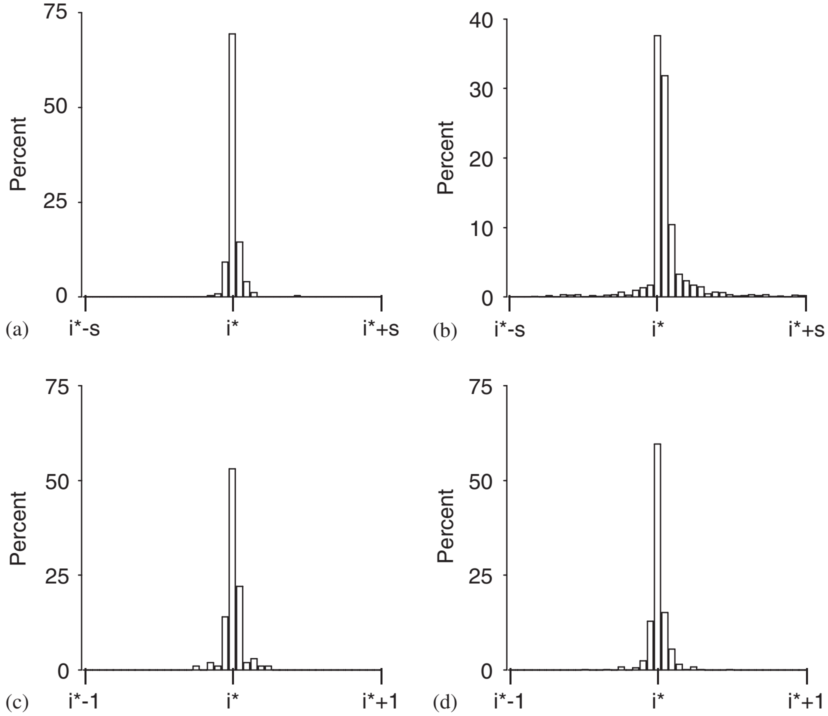

Fig. 5. Market rates in the U.S. and Europe (relative to the central bank's target): (a) daily fed funds, (b) daily Eonia. Note: Graph a is based on daily effective fed funds rates, June 2003–June 2004, when $i^* = 1$. Graph b is based on the daily eonia from April 1999 to December 2004 with varying i^*. In each case, $s = 1$ percentage point. Intraday federal funds rates relative to target, (c) June 2003–June 2004, and (d) June 2002–June 2003. Note: For graph c, the Fed's target funds rate was 1 percent. For graph d, the target funds rate was 1.75 percent until November 2002, then 1.25 percent.

returns on alternative investments'' (Federal Reserve Board, 1998). The Fed has stated that ''greater efficiencies and regulatory burden reduction might be realized by substantially reducing, or even eliminating, reserve requirements … [With] explicit interest on contractual clearing balances or a similar type of voluntary instrument maintained over a two-week average period … monetary policy [could potentially] be implemented effectively'' (Kohn, 2004). Moreover, as noted above, the BOE now intends to implement a system of voluntary reserve commitments. The final model of the paper investigates key features of such regimes.

4. Voluntary reserve commitments

Suppose the social cost of an overnight overdraft, which involves a commitment of financial resources and credit risk for a central bank, is larger than the social cost of a deficiency on a bank's commitment to hold period-average balances, which involves only a marginal effect on the central bank's ability to smooth overnight interest rates. This ranking of social costs reflects the structure of penalties at the Fed and the Bank of Japan, for instance.[7] If period-average commitments paid more than overnight excess balances, a bank might then voluntarily choose to establish a reserve commitment as a precaution against running overdrafts. In the U.S., for instance, numerous banks with no need to hold balances in their Fed account (because sweep programs have eliminated their compulsory requirements) voluntarily commit to hold contractual clearing balances, which earn implicit interest, unlike excess reserves, and also protect against overdrafts.[8]

Below then, a tractable one-day model of a voluntary reserve commitment regime is developed under the following assumptions:

(A6) Overdrafts and other borrowings cost i^*+w and count toward meeting reserve commitments. Shortfalls in meeting commitments are subject to a penalty of v. Reserve commitments earn i^*-v and excess reserves earn i^*-s, where $0<v<s$.

These assumptions create symmetric opportunity costs around the target rate of meeting or falling short on reserve commitments, equal to v. As shown below, this is the crucial symmetry in this model. Unlike the rate corridor regimes studied above, it is irrelevant whether the borrowing and lending spreads, w and s, are equal. Indeed, the main results hold even if the interest rate on excess reserves is zero ($s = i^*$).[9]

A representative bank's expected costs can be written as a function of the reserve commitment, R, which may be zero. Again, i is the known market rate when the target balance T is chosen:

$$\begin{aligned} C(R) = \min_T\,& (i - i^* + v)R[1 - F(R - T)] + \int_{R-T}^{\infty} (i - i^* + s)(T + \varepsilon - R)\mathrm{d}F(\varepsilon) \\ & - (i^* + w - i)\int_{-\infty}^{-T} (T + \varepsilon)\mathrm{d}F(\varepsilon) + vRF(-T) \\ & + \int_{-T}^{R-T} v(R - T - \varepsilon)\mathrm{d}F(\varepsilon) + (i - i^* + v)\int_{-T}^{R-T} (T + \varepsilon)\mathrm{d}F(\varepsilon). \end{aligned} \quad (11)$$

The first line of Eq. (11) gives expected costs when the account shock puts the bank in an excess reserves position; the first and second terms being the opportunity costs of reserve commitments and excess reserves, respectively. The second line represents overdraft situations: the first-term is the opportunity cost of borrowing to bring the account balance to zero and the second term is the reserve deficiency charge. The third line gives the situation when the account balance partially fulfills the reserve commitment: the first-term is the deficiency charge on the shortfall and the second term is the opportunity cost on the reserve balances that are held. The first order condition is

$$i - i^* + s(1 - F(R - T^*)) - wF(-T^*) = 0. \quad (12)$$

[7]The Fed's charge for overnight overdrafts is 4 percentage points above the effective federal funds rate, while reserve deficiencies cost only 2 percentage points above the target funds rate and contractual clearing balance deficiencies are charged a fixed 2 percent rate. At the Bank of Japan, overdrafts cost 6 percentage points above the discount rate, while reserve shortfalls cost 3.75 percentage points above that rate. At the ECB, however, overdrafts are charged the marginal lending rate, while reserve shortfalls incur a premium of 2.5 percentage points. See Blenck et al. (2001).

[8]The ECB has a similar ranking of payment rates; it offers the marginal lending rate on period-average required reserves and 100 basis points less on its daily deposit facility.

[9]A central bank could create an equivalent symmetry by charging a facility fee of vR, paying the target rate of i^* on balances held up to the contractual maximum of R, and assessing no penalty for shortfalls (aside from overdrafts). In this case, R would be seen as a maximum balance that could earn i^* rather than a commitment.

The response of reserve demand to a higher commitment is:

$$\frac{\partial T^*}{\partial R} = \left[1 + \frac{w}{s}\frac{f(-T^*)}{f(R - T^*)}\right]^{-1} > 0 \quad \text{and} \quad < 1. \tag{13}$$

To determine the bank's optimal choice of a reserve commitment, in principle, the optimal T^* from Eq. (12) would be substituted into Eq. (11) to obtain a cost function, and the unconditional expectation of that cost function (across the distribution of market interest rates) would be taken. Finally, the optimal value of R would be found by differentiating that unconditional cost function. For convenience, the derivative of Eq. (11) is taken with respect to R first. The first-order condition for R is

$$\mathrm{E}_{I_u}\left(\frac{\partial C}{\partial R}\right) = \mathrm{E}_{I_u}(v + s[F(R^* - T^*) - 1]) = 0, \tag{14}$$

where T^* is also a function of R^*, given by Eq. (12), and I_u is the unconditional information set. Eq. (14) is a complex nonlinear function whose solution would depend on the bank's perceived distribution of the market interest rate. That distribution could have a variety of shapes, as suggested by Fig. 2, reflecting in part the supply of non-borrowed reserves by the central bank. However, some conclusions may be drawn without specializing the model much further. First, with T^* in Eq. (12) independent of v, given R, the comparative static result can be written as

$$\frac{\partial R^*}{\partial v} = -\left[\mathrm{E}_{I_u}\left(sf(R^* - T^*)\left[1 - \frac{\partial T^*}{\partial R^*}\right]\right)\right]^{-1} < 0, \tag{15}$$

using Eq. (13). Now assume merely that banks expect the market interest rate to equal on average the central bank's desired rate: $I_u = \{F(\varepsilon), \quad \mathrm{E}(i) = i^*\}$.

Solutions for R depend on the value of v. First, two extremes can be identified. If the opportunity cost of reserve commitments were equal to that of excess reserves ($v = s$), Eq. (14) would require $F(R^* - T^*)$ to equal zero, implying an optimal reserve commitment of $-\infty$. Next, observe that condition (14) can be written in an alternative form by substituting for $F(R^* - T^*)$ from Eq. (12):

$$\mathrm{E}_{I_u}\left(\frac{\partial C}{\partial R}\right) = \mathrm{E}_{I_u}(v + i - i^* - wF(-T^*)) = 0. \tag{16}$$

If there were no expected opportunity cost of a reserve commitment ($v = 0$), Eq. (16) could be satisfied only if $F(-T^*)$ equaled zero, which would imply $T^* \to \infty$ requiring $R^* \to \infty$.

Finally, the existence of interior solutions for R^* can be demonstrated:

Proposition. For small positive values of v, the optimal choice of a reserve commitment in problem (11) is positive and finite.

Proof. First note that the second-order condition for R^* to be a minimum holds:

$$\mathrm{E}_{I_u}\left(f(R^* - T^*)\left[1 - \frac{\partial T^*}{\partial R^*}\right]\right) > 0,$$

using Eq. (13). Next, note that, when $R = 0$, the left-hand side (LHS) of Eq. (14), after substituting for $F(0 - T^*)$ from Eq. (12) and taking expectations, is negative when

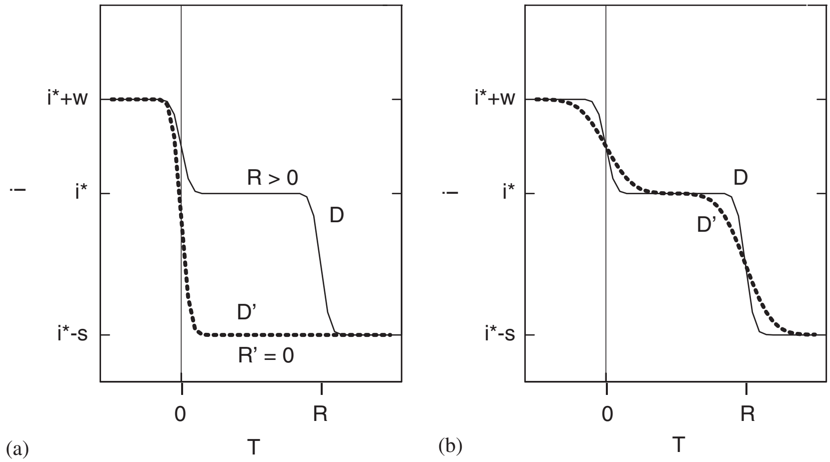

Fig. 6. Voluntary reserve commitments. Demand for central bank balances: (a) with/without reserve commitment, and (b) increase in uncertainty.

$v < sw/(s+w)$. Thus, for positive values of v below this threshold, Eq. (14) can only be satisfied with an R^* greater than zero. Finally, because $R-T^*$ is, from Eq. (13), increasing in R, so is $F(R-T^*)$. As $R\to\infty$, the LHS of Eq. (14) goes to v. Thus, the LHS of Eq. (14) must hit zero at a finite positive value for the voluntary reserve commitment. □

Demand for central bank balances in this model, assuming a normal distribution, is graphed in Fig. 6. The demand curve without a reserve commitment, given by the dotted line in Fig. 6(a), is similar to that in Fig. 1. Demand for balances equals zero at the midpoint between the interest rates on overdrafts and on deposits at the central bank, which is depicted to be below i^* in the given example. With a positive reserve commitment, the solid line, the demand curve has a highly elastic region at i^*. This elasticity does not depend on the relative values of w and s, nor on any market expectation for the interest rate, but only on the symmetric opportunity costs around the target rate of meeting or missing the reserve commitment.[10] When the market rate is close to the target rate, risk-neutral banks are relatively indifferent between buying funds in the market or falling short on their commitment. By contrast, in the multi-day model of the previous section, the elasticity induced *around* average daily-required reserves on a pre-settlement day was attributable to intertemporal arbitrage.

As suggested by Fig. 6(b), the size of the chosen reserve commitment relative to the account balance uncertainty determines the width of the elastic region, but not its location. The central bank could control the optimal choice of reserve commitment by adjusting the spread v, as indicated by Eq. (15). Presumably, the central bank would set the spread to ensure that it had a sufficient period-average demand for the smooth implementation of monetary policy without absorbing too many funds from the banking system.

A reserve demand curve with a highly elastic region on settlement day would be of great help to a central bank in hitting its interest rate target. Under existing policy

[10]Examination of Eq. (12) reveals that a condition similar to that shown in Eq. (10) holds near i^*.

implementation procedures, interest rates tend be much more volatile on reserve settlement days than on pre-settlement days, as noted by Hamilton (1996) and others. The settlement-day elasticity generated by the model of course depends on the ability of a central bank to create symmetric opportunity costs, which could be undermined, for instance, by non-pecuniary penalties such as disapproval from bank examiners when reserve deficiencies occur. Such considerations would argue for contracts in which R is the maximum balance earning the indicated rate, rather than a commitment *per se*.

5. Concluding discussion

For most major central banks, the volatility of overnight interest rates has been rather subdued in recent years, suggesting that a range of policy implementation frameworks have performed fairly well. However, the generally stable and low level of interest rates of late has likely contributed to the modest level of volatility. In periods when a central bank's target interest rate is higher and more variable, volatility issues may be of heightened concern, and a best practice policy implementation framework could be a linchpin in a central bank's ability to smooth interest rates around its target. Among the available choices for such a framework are reserve and rate corridor regimes, hybrid systems, and regimes with voluntary reserve commitments.

The symmetry of opportunity costs in a rate corridor gives a central bank knowledge of the quantity of reserves demanded at its target rate. However, perfect substitution of central bank loans and deposits with market transactions is less likely in more complex money markets, and if symmetry is broken, other weaknesses of rate corridor regimes are more evident, including less elastic demand at the central bank's target rate. Central banks employing a rate corridor would be well advised to remove frictions that impair substitutability, such as differential interest rates for overdrafts versus other loans. In addition, automatic payment of interest on account balances is preferable to a deposit facility that forces banks to intervene to shift funds into interest-earning accounts.

The above models and the experience to date suggest a continuing advantage for period-average reserves, even in the presence of a rate corridor. Period-average requirements create incentives for intertemporal arbitrage resulting in an elastic region in the demand for central bank balances on pre-settlement days around the expected interest rate on settlement day. Traditionally, period-average reserve demand has been arranged through a system of reserve requirements. However, a central bank tends to lose control of a reserve requirement system over time as banks invent new means of avoiding the reserve tax, and the resources devoted to such activities may impair efficiency.

The best practice policy implementation framework for the future may include elements of both interest rate corridors and period-average reserves. The final model of this paper pointed in that direction by analyzing a system of voluntary reserve commitments. It also depicted symmetric opportunity costs of meeting or falling short of reserve commitments, a feature that could smooth interest rates on settlement day, but has not yet been employed by central banks.

The Federal Reserve's current program of contractual clearing balances offers a prototype for some aspects of a voluntary commitment regime. Banks with no reserve balance requirement at the Fed do now voluntarily commit to hold period-average balances, and such commitments offer the same interest-smoothing benefits as reserve requirements. However, contractual clearing balances were designed primarily to help banks manage their Fed accounts without incurring overdrafts rather than as an aid to policy implementation by the Open Market Desk. For the latter purpose, the program is flawed because of the absence of Congressional authorization for explicit interest payments. Because contractual balances can earn credits only to defray charges for Fed services, the Fed does not control the overall level of such balances for the purpose of smoothing interest rates. And when interest rates rise, the level of such balances can fall sharply, as banks earning credits to cover a constant level of service charges have unit elastic demand.

The BOE is planning to move to a system of one-month voluntary reserve commitments, which unlike the Fed's contractual clearing balance program, will be designed and controlled for the purpose of monetary policy implementation. The BOE currently operates a rate corridor system, but has complained that "the current operational framework leaves sterling overnight rates considerably more volatile than is desirable" (Bank of England, 2004). The BOE will try to avoid rate volatility on settlement day by shrinking its rate corridor to 25 basis points on either side of its target rate on that day, compared with 100 basis points on other days, and also by giving banks a penalty-free band of 2 percent around their reserve commitment. A penalty-free band is also a feature of the Fed's contractual clearing balance program; it is less costly to administer than the carryover feature of the Fed's reserve requirement system that is also designed to limit settlement-day volatility.

The third model of this paper suggests that there are other creative mechanisms that central banks could explore to address the issue of settlement-day volatility. In particular, while central banks have recognized the importance of symmetry in their loan and deposit rate spreads by establishing rate corridor regimes, they have not as yet designed regimes with symmetric costs of meeting or falling short of period-average reserve requirements or voluntary reserve commitments. The BOE, for instance, intends to pay its target (repo) rate on reserve commitments, implying no opportunity cost, while shortfalls cost twice that rate. Symmetric opportunity costs of meeting or falling short on reserve commitments would instead provide the key advantage of an elastic reserve demand curve around the desired overnight interest rate on the last day of a maintenance period.

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